Design and Analysis of Laminated Composite Plates using MATLAB

Prashanth M D, Vineeth Kumar T V, Kartik

Abstract— This paper discusses the response of simply supported, symmetrically laminated composite plates subjected to a uniformly distributed load. ANSYS is used to both model and perform Finite Element Analysis (FEA) on the plate of a 4-ply composite at first. The response of the plate to the uniformly distributed load was analyzed and validated through calculations using MATLAB in coherence with the classical lamination theory. Once the FEA had been validated, an investigation on the effects of the fiber orientation to the displacement value was conducted for different cases of symmetric 4-ply layup. The same analysis was completed and validated for a 20-ply plate and in addition will reveal the effect of stacking sequence on bending stiffness.

Index Terms— Composites, Laminate, Analysis, UDL, MATLAB, FEA.

1 Introduction

Composites are one of the most advanced and adaptable known engineering materials. Progresses in the field of materials science and technology have given birth to these fascinating and wonderful materials. Composites are heterogeneous in nature, created by the assembly of two or more components with fillers or reinforcing fibres and a compactable matrix [1]. The matrix may be metallic, ceramic or polymeric in origin. It gives the composites their shape, surface appearance, environmental tolerance and overall durability while the fibrous reinforcement carries most of the structural loads thus giving macroscopic stiffness and strength [2]. A composite material can provide superior and unique mechanical and physical properties because it combines the most desirable properties of its constituents while suppressing their least desirable properties.

At present composite materials play a key role in aerospace industry, automobile industry and other engineering applications as they exhibit outstanding strength to weight and modulus to weight ratio. High performance rigid composites made from glass, graphite, Kevlar, boron or silicon carbide fibres in polymeric matrices have been studied extensively because of their application in aerospace and space vehicle technology [3-8].

Fiber-reinforced plastics (FRP) have an extensive array of applications. They range from structural to recreational use. The aerospace and automotive industries look to composites to improve fuel economy due to its high strength to weight ratio. The sports industry looks to composites to improve sports equipment technologies.

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The fact that composites offer increased strength without sacrificing additional weight is what gives composites the advantage from most structural and recreational materials. FRP's are often manufactured in laminates. A laminate consists of individual lamina or plies as seen in Figure 1. These plies consist of different combinations of fiber materials embedded in a matrix material, usually of a polymer resin.

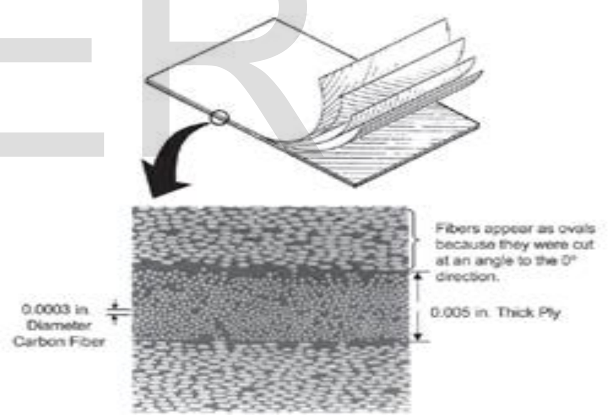

Fig 1: Cross-section of cross-ply laminate[2]

When designing for the use of composites, there are many aspects to investigate as there are many variables that affect a laminate's response to load. Fiber and matrix material, fiber orientation, layer stacking sequence are some of the variables affecting the response of a laminate. The virtually limitless combinations of ply materials, ply orientations and ply-stacking sequences increase the design flexibility of composite structures. In this paper the response of composite plates to uniformly distributed load is investigated.

2 Methodology

In this study FEA models of a simply supported, symmetric, composite laminate plate under a uniformly distributed load are employed. In the first step FEA model of a composite plate that consists of 4 layers is developed. The model will be subjected to a uniformly distributed load. Deflection results will be analyzed and validated with hand calculations using MATLAB. The classical lamination theory is used to arrive at a deflection value of the plate. Once the hand calculations validate the 4-ply FEA model, an investigation of the effects of fiber orientations on deflection results will be conducted. Deflection results are then discussed, and a case of a 20-ply laminate, symmetric plate will be modeled and analyzed. This model will also be validated through hand calculations with investigations on how additional plies and layup sequence affects the deflection results.

3 Governering Equations For The Response Of A Laminated Plate

The governing equations consist of the behavior of the plate internally as well as the behavior of the boundary conditions. The governing equations are derived using the Newtonian approach where summing the forces and moments on the plate is used to develop the differential equations. Figure 2 shows a composite plate in the x-y-z orientations, loading of q (x, y) and a width 'a' and a length 'b' with the reference point at the center of the plate.

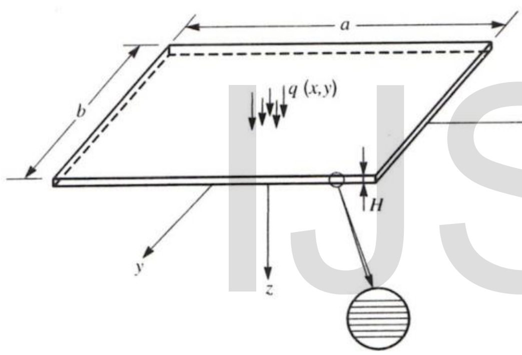

Fig 2: Geometry, Nomenclature and Loading of the plate

Equations (1) - (3) are in terms of displacements that govern the response of a composite plate. In this work, investigations are confined to a simply supported, symmetrically laminated composite plate.

$$A_{11}\frac{\partial^2 u^o}{\partial x^2}+2A_{16}\frac{\partial^2 u^o}{\partial x\partial y}+A_{66}\frac{\partial^2 u^o}{\partial y^2}+A_{16}\frac{\partial^2 v^o}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 v^o}{\partial x\partial y}+A_{26}\frac{\partial^2 v^o}{\partial y^2}$$
$$-B_{11}\frac{\partial^3 w^o}{\partial x^3}-3B_{16}\frac{\partial^3 w^o}{\partial x^2\partial y}-(B_{12}+2B_{66})\frac{\partial^3 w^o}{\partial x\partial y^2}-B_{26}\frac{\partial^3 w^o}{\partial y^3}=0 \quad (1)$$

$$A_{16}\frac{\partial^2 u^o}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 u^o}{\partial x\partial y}+A_{26}\frac{\partial^2 u^o}{\partial y^2}+A_{66}\frac{\partial^2 v^o}{\partial x^2}+2A_{26}\frac{\partial^2 v^o}{\partial x\partial y}+A_{22}\frac{\partial^2 v^o}{\partial y^2}$$
$$-B_{16}\frac{\partial^3 w^o}{\partial x^3}-(B_{12}+2B_{66})\frac{\partial^3 w^o}{\partial x^2\partial y}-3B_{26}\frac{\partial^3 w^o}{\partial y^3}=0 \quad (2)$$

$$D_{11}\frac{\partial^4 w^o}{\partial x^4}+4D_{16}\frac{\partial^4 w^o}{\partial x^3\partial y}+2(D_{12}+2D_{66})\frac{\partial^4 w^o}{\partial x^2\partial y^2}+4D_{26}\frac{\partial^4 w^o}{\partial x\partial y^3}+D_{22}\frac{\partial^4 w^o}{\partial y^4}$$
$$-B_{11}\frac{\partial^3 u^o}{\partial x^3}-3B_{16}\frac{\partial^3 u^o}{\partial x^2\partial y}-(B_{12}+2B_{66})\frac{\partial^3 u^o}{\partial x\partial y^2}-B_{26}\frac{\partial^3 u^o}{\partial y^3}-B_{16}\frac{\partial^3 v^o}{\partial x^3}$$
$$-(B_{12}+2B_{66})\frac{\partial^3 v^o}{\partial x^2\partial y}-3B_{26}\frac{\partial^3 v^o}{\partial x\partial y^3}-B_{22}\frac{\partial^3 v^o}{\partial y^3}=0 \quad (3)$$

4 FEM Modelling

The SHELL181 element was chosen since input parameters of a composite plate can be easily modified due to it being a shell element and also it is suitable for thin to moderately thick shell structures and commonly used for laminated composite shells. SHELL181 is a 4-node element with six degrees of freedom at each node.4 Figure 3 shows its geometry.

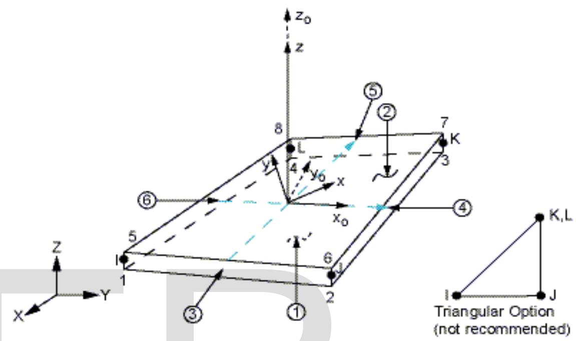

Fig 3: SHELL181 Geometry4

VM82 analyzes a [0 90] $_S$, 5m by 5m composite laminate plate subjected to a uniformly distributed load in the transverse direction with the defined material properties.
Figure 4 shows a visual representation of the boundary constraints applied in ANSYS.

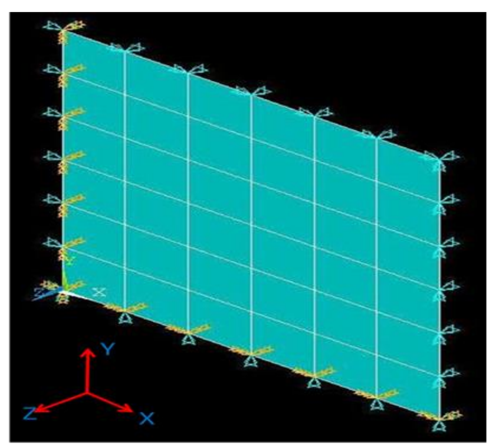

Fig 4: Visual Representation of Boundary Conditions

5 Results

The results pertaining to the analyses run through MATLAB and the FEA through ANSYS are discussed in this section.

5.1 4-Ply laminate

Table 1 shows the results of not only analyses run for a [0 90]S but for a symmetric angle-ply plate in which the inner layers are kept at 90º while the fiber orientation of the outer layers are increased ranging from 0º to 90º. The bending stiffness values for D16 and D26 were computed in the MATLAB. The MATLAB code outputs the [D] matrix where the D16 and D26 values were acquired.

TABLE I

ANSYS AND MATLAB RESULTS FOR 4-PLY COMPOSITE PLATES

Layup	ANSYS Deflection (m)	MATLAB Deflection (m)	% Error	D16 (N-m)	D26 (N-m)
[0 90]s	0.0681	0.0680	0.1468	0.0000	0.0000
[10 90]s	0.0883	0.0626	29.1053	288.1000	12.0000
[20 90]s	0.1078	0.0523	51.4842	493.4000	70.4000
[30 90]s	0.1164	0.0440	62.1993	565.8000	193.8000
[40 90]s	0.1160	0.0399	65.6034	505.3907	358.4758
[45 90]s	0.1154	0.0393	65.9445	438.5965	438.5965
[50 90]s	0.1155	0.0398	65.5411	358.4758	505.3907
[60 90]s	0.1165	0.0436	62.5751	193.8000	565.8000
[70 90]s	0.1100	0.0514	53.2727	70.4000	493.4000
[80 90]s	0.0889	0.0605	31.9460	12.0000	288.1000
[90 90]s	0.0651	0.0650	0.1536	0.0000	0.0000

Figure 5 is the graphical output of the ANSYS FEA result for a [0 90]S plate. The graphic as well as the result in Table 1 shows that the MATLAB code is validated through the FEA result. The MATLAB code resulted in a 0.0680m deflection while the FEA resulted in 0.0681m deflection, a 0.14% error.

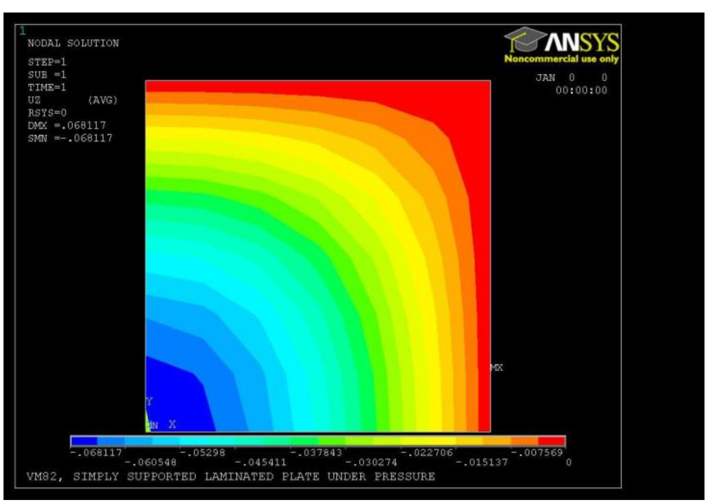

Fig 5: Deflection of 4-Ply [0 90] S Plate

5.1 4-Ply laminate

The values of D16 and D26 become small when a large number of plies are stacked and become insignificant for thicknesses of more than sixteen plies [1]. This section investigates how the values of D16 and D26 become insignificant when the number of plies is increased to more than sixteen plies. FEA analysis and MATLAB calculations were conducted for 20-ply plates. These plates consist of the same material properties and same loading is applied. Thickness of each layer however was reduced from 0.025m to 0.0025m to keep the requirement of the classical lamination theory where thickness is ten times smaller than the length or width of the plate. The results of the analyses are tabulated in Table 2.

TABLE II

ANSYS AND MATLAB RESULTS FOR 20-PLY COMPOSITE PLATES

Layup	ANSYS Deflection (m)	MATLAB Deflection (m)	% Error	D16 (N-m)	D26 (N-m)
[60 0 90 90 0 90 90 0 90 0] s	0.5829	0.4698	19.4030	7.5042	21.906
[0 60 90 90 0 90 90 0 90 0] s	0.5733	0.4842	15.5416	6.0089	15.195
[0 0 60 90 0 90 90 0 90 0] s	0.5657	0.4970	12.1442	4.6797	13.660
[0 0 90 60 0 90 90 0 90 0] s	0.5606	0.5093	9.1509	3.5167	10.265
[0 0 90 90 60 90 90 0 90 0]s	0.5576	0.5208	6.5997	2.5199	7.3559
[0 0 90 90 0 60 90 0 90 0] s	0.5547	0.5300	4.4529	1.6891	4.9309
[0 0 90 90 0 90 60 0 90 0] s	0.5533	0.5379	2.7833	1.0246	2.9909
[0 0 90 90 0 90 90 60 90 0] s	0.5525	0.5440	1.5385	0.5261	1.5358
[0 0 90 90 0 90 90 0 60 0] s	0.5519	0.5481	0.6885	0.1938	0.5658
[0 0 90 90 0 90 90 0 90 60] s	0.5517	0.5501	0.2900	0.0277	0.0808

6 CONCLUSION

An investigation on the response of a symmetric composite laminate plate was conducted. ANSYS FE package was utilized to model and simulate the response of a simply supported symmetric plate subjected to a uniform load. VM82 was used for the purposes of this project because it is a solved simply supported, symmetric plate problem. It was used to validate the hand calculations performed using MATLAB that invoked the classical lamination theory and modified accordingly to analyze the different cases. The MATLAB solutions along with the ANSYS analyses were performed to investigate how fiber orientation, lay-up sequencing and total number of plies affect the deflection results.

The response of a symmetric 4-ply plate with a layup of [0 90] S was performed using VM82 to validate the MATLAB code. The effect of fiber orientation of the outer plies of the 4-ply plate was then investigated.
The following conclusions are drawn:
1. The minimum deflection of the 4-ply plate is when the lay-up is [90 90] S
2. The maximum deflection occurs when the outer layers of are oriented at 60º
3. The large % error between the MATLAB and ANSYS deflection results is due to values of D16 and D26 being non-zeros for plies whose angles are those other than 0º or 90º, thus changing the governing differential equation.

The response of a symmetric 20-ply plate with an original layup up of [0 0 90 90 0 90 90 0 90 0]S was conducted to investigate how an increase in the number of plies affects the deflection results.

The following conclusions are drawn for 20-ply plate:

1. The % error between the MATLAB and ANSYS deflection results has decreased when sixteen plies were added to the original four. This is due to the decrease of D16 and D26 values in Table 2. These values are significantly smaller than those in Table 1 when there were only four plies to the plate.
2. As the 60º ply was re-located from the outer plates to the inner plates, the stiffness of the plate increased. The minimum deflection in Table 2 is when the 60º ply was in layers ten and eleven, straddling the midplane.
When designing for composite materials with a specific purpose, there are several variables an engineer should investigate. Fiber and matrix material, fiber orientation, layer stacking sequence and so on affect the response of a laminate. The virtually limitless combinations of ply materials, ply orientations and ply-stacking sequences increase the design flexibility of composite structures.

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